New Methods to Determe Gravity Probe-B Spin Parameters using Graz kHz SLR Data

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Abstract

Using kHz data of the SLR station Graz, spin parameters of the satellite Gravity Probe B (GP-B) are derived; these include spin period and its change over a 1.5 year period, as well as spin direction, and spin axis orientation. The results are compared to the actual data sets - as determined by the GP-B mission itself – thus allowing independent confirmation of the kHz SLR derived results.

Introduction

GP-B was launched on April 20th, 2004, into a polar orbit at 640 km altitude. During its measurement phase, the spacecraft was spinning slowly - with about 77.5 seconds / revolution - around its central axis, defined by a telescope at one end, and the laser retro reflector (LRR) array at the other end. Its orientation was maintained always to point with high accuracy to the star IM-Pegasus; the direction to this star is measured with the on-board telescope with a stability of 0.1 milliarcseconds per year [1] (ed.).

The LRR array (Fig. 1) on GP-B consists of 8 retro reflectors in a ring-like formation, and a central LRR [2]. While such an arrangement only spreads standard SLR measurements, the high resolution of kHz SLR allows to scan the single reflectors, to identify their motion due to the spin of the satellite, and to derive all GP-B spin parameters from kHz SLR data.

Spectral Analysis of kHz Slr Data

The spectral analysis of kHz SLR data is based on residuals obtained by subtracting the calculated, predicted orbit, from the measured distances. Fitting a low order polynomial to these residuals allows elimination of outliers, but keeps the oscillating signal of the eight rotating LRR's (Fig. 2, top).

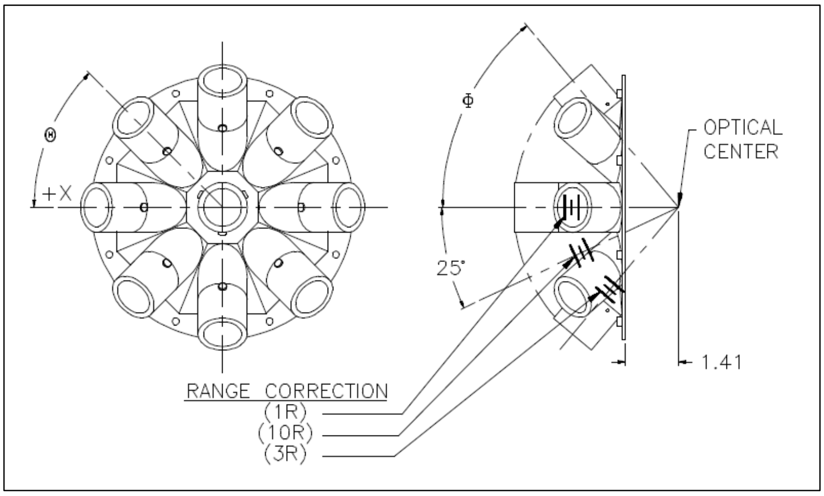


Fig. 1: *GP-B Laser Retro Reflector (LRR) Design*

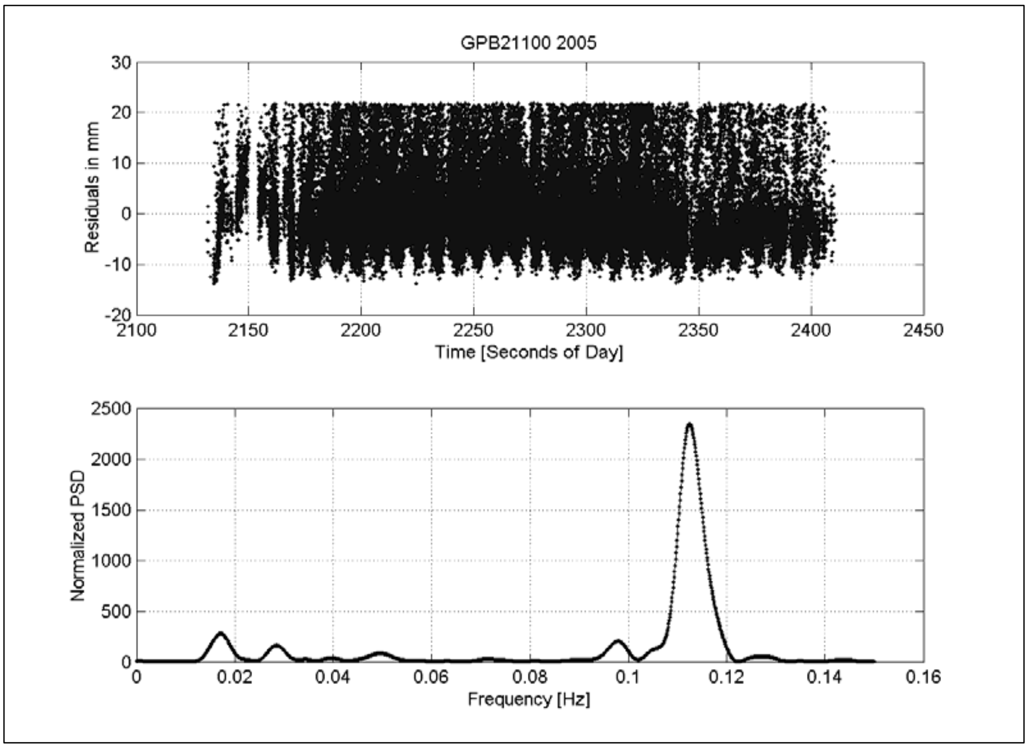


Fig. 2: *Residuals of a 280 seconds segment of a GP-B pass of DOY 211/2005 (top); frequency spectrum generated by these residuals (bottom).*

The Lomb method of spectral analysis was suggested in [3] alternatively to the Fast Fourier Transform (FFT), allowing for non-equally spaced data, as it is the case for such SLR measurements. The FFT could still be used if the data gaps were interpolated, but this would introduce unwanted frequencies. Therefore the Lomb method was preferred.

Taking into account the known inertial spin period of GP-B (77.5 seconds per revolution) during phase A (Fig. 3), and the 8 retro reflectors per revolution, we selected passes with at least 100 seconds to analyze a minimum of 10 oscillations, to get reliable results for the spectral power (Fig. 2, bottom).

This spectral power varies from pass to pass, with the data gaps and the length of the pass being the main corrupting factors. The analysis has been performed also on selected intervals of the longer passes, with high data density, as an additional verification of the frequency obtained for the complete pass.

Spin Period Trend

From all GP-B passes measured by Graz kHz SLR, we selected those with more than 50,000 returns per pass. Applying the Lomb analysis to these passes, we found three different regions of spin periods after the initialization period, as soon as SLR measurements started (Fig. 3, top): phase A: from 10.08.2004 until 6.09.2005, the mean spin period was about 77.5 seconds; phase B: the spin period changed to about 125 seconds; after 11.01.2006 (phase C), the spin period analysis shows an unstable behavior, as expected after termination of the active phase of the GP-B experiments (Fig. 3, top). Comparing the SLR derived spin periods with the GP-B based data set for phase A (Fig. 3, bottom), the RMS of the differences is 4.99 seconds.

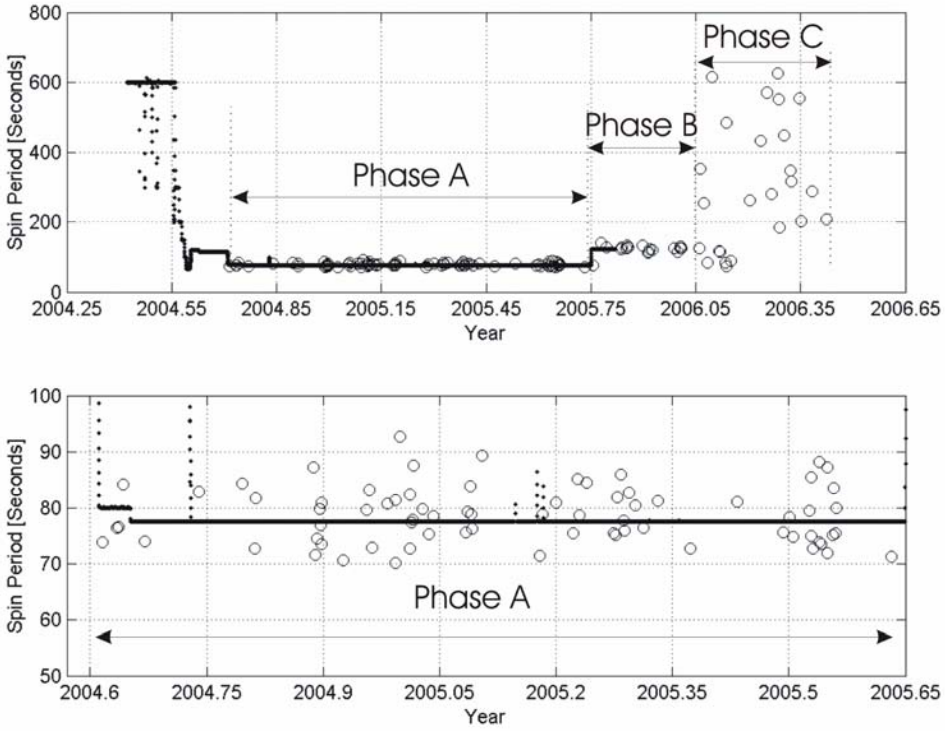


Fig. 3. *GP-B spin period variations. Dots indicate spin periods as measured on-board; circles (o) show results of kHz SLR spectral analysis. Bottom: Expanded view for phase A, showing RMS of 4.99 seconds.*

Apparent Spin

Although the spectral analysis already yields clear results – coinciding with the GP-B on-board measured data sets (Fig. 3) -, the accuracy is not as expected: the frequency peak (Fig. 2, bottom) is well defined, but rather broad; and the RMS of the differences between kHz SLR based periods and the on-board spin measurements (Fig. 3) amounts to rather high 4.99 seconds for phase A.

Simulating the measured GP-B passes, using all known parameters (GP-B orbit, Earth rotation, fixed pointing of GP-B to IM-Pegasus, inertial GP-B spin period as measured by the spacecraft itself, geometry of the retro reflectors, as well as their range corrections, etc.), the influence of the apparent spin - the satellite's spin as observed from Earth – was identified as the main reason (Fig. 4). GP-B's spin period is about 77.5 seconds; because the satellite moves along its orbit considerably during this time, the apparent spin period for even the short part (151 seconds) of the pass in Fig. 4 changes from initial 72.8 seconds (9.1 x 8 retro reflectors) to 70.4 seconds (as determined from peak-to-peak distances; Fig. 5). This change in apparent spin period is the main reason for the mentioned inaccuracies in the spectral analysis. In addition, the change of the incident angle of the laser beam causes a decrease of the "modulation depth", as indicated by the line in Fig. 4.

Spin Period Determination Using Simulation

Due to the low spin rate of GP-B, it is not possible to apply the apparent spin directly to the spectral analysis results, as it has been done in [4]; we therefore checked other

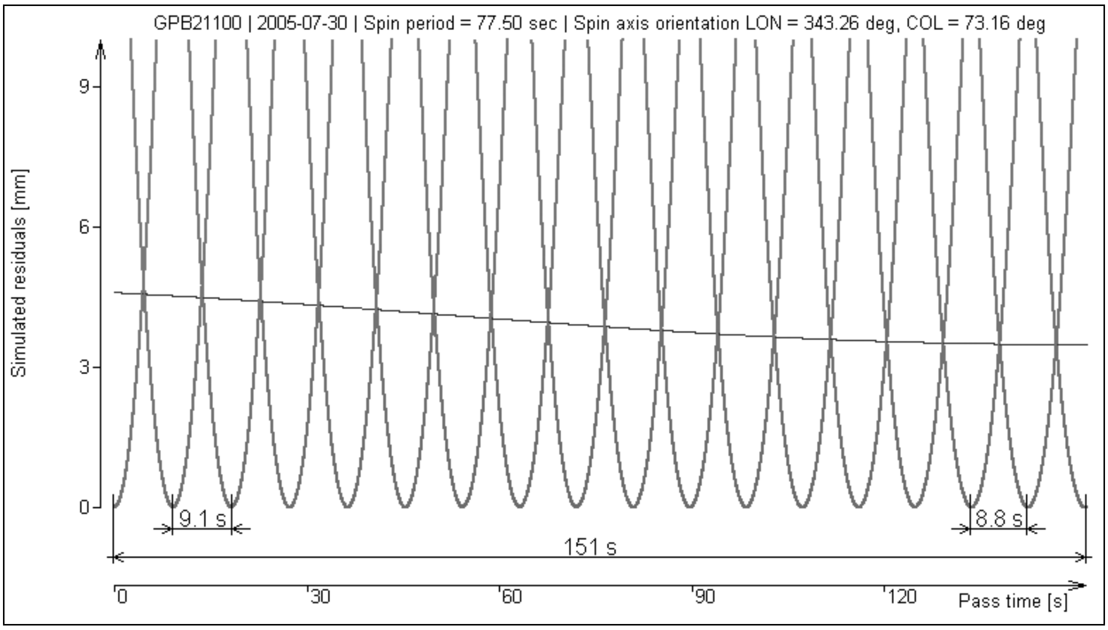


***Fig. 4:.** Simulation of GP-B pass DOY 211/2005; spin period slightly changing due to apparent spin. The line shows the decreasing "modulation depth" during the 151 seconds.*

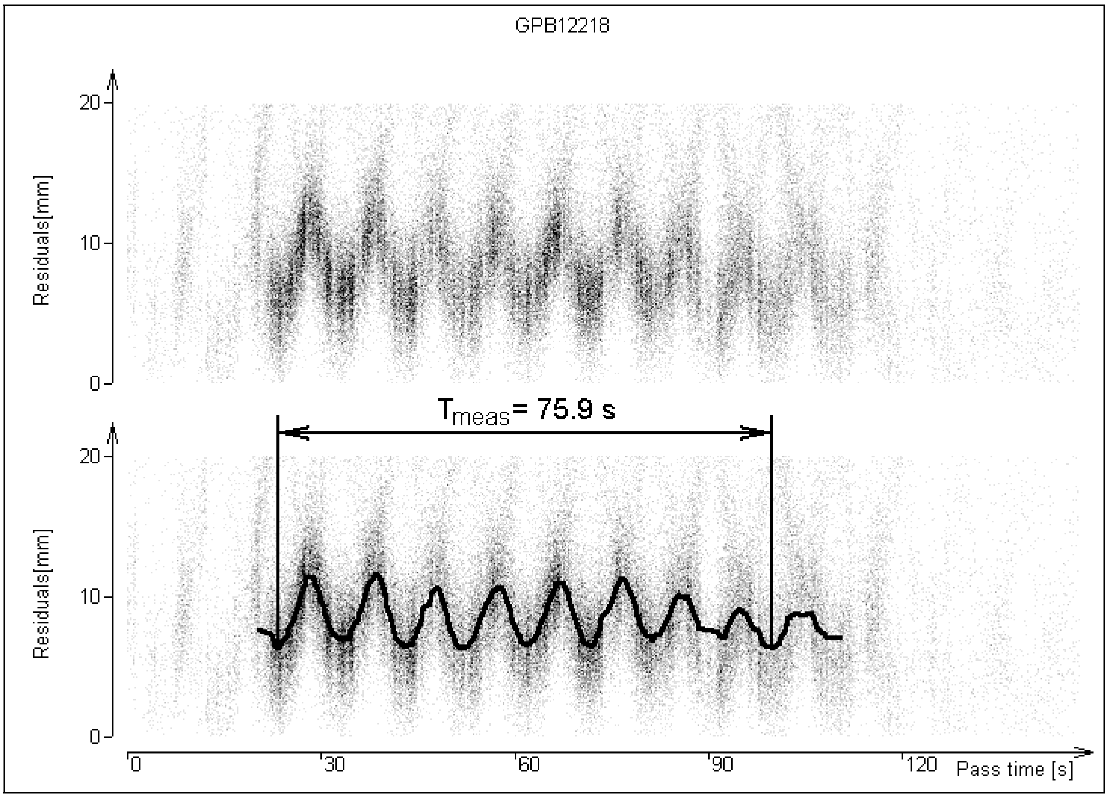


Fig. 5a: *GP-B pass of DOY 122 / 2005, Top: Residuals; about 79200 points in 151 s; bottom: solid line: averaging; 75.9 s from first to last peak (T_{meas}).*

methods to calculate more accurate, inertial spin periods for GP-B using our kHz SLR data.

The simulations, as described above, proved to be a good and powerful tool: for each measured pass, we determined the time period from first to last peak (Fig. 5a, T_{meas}); the same pass was simulated also (Fig. 5b, T_{sim}); however, the inertial spin period of

GP-B was used here as parameter, varying its value from –50 to –100 seconds, and from 50 to 100 seconds, in steps of 0.01 seconds. If the estimated and the true inertial spin periods coincide, the measured and the simulated T values for the same epoch times should be the same. In Fig. 6, the differences T_{meas} - T_{sim} for 100 phase A passes are plotted, allowing for both spin directions. The zero-crossings of these differences determine the inertial GP-B spin periods.

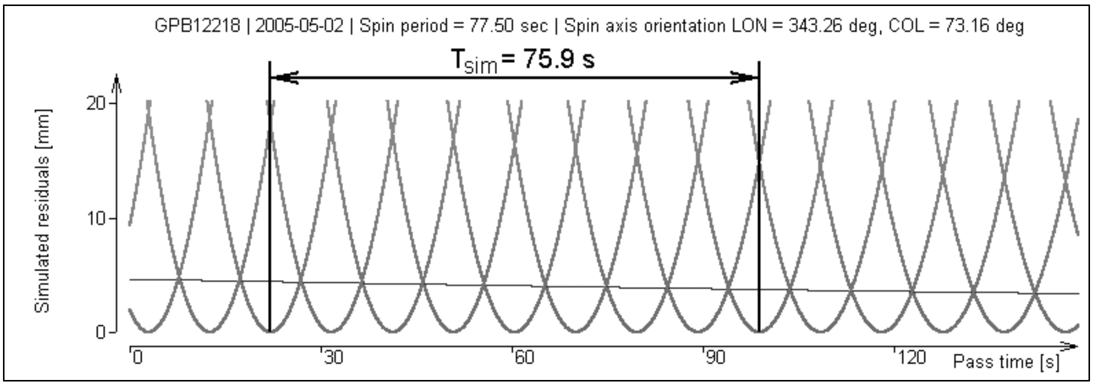


***Fig. 5b**: Simulation of same pass of DOY 122 / 2005: T_{sim} is same as T_{meas} at same epoch time, when simulating with inertial spin period of 77.50 seconds.*

Applying this method to 86 GP-B passes of phase A (selected to contain at least 5 peaks), the resulting spin period values coincide well with spin data as measured by GP-B (Fig. 7a); the accuracy of the resulting inertial spin period is improved now, with an RMS value of 0.98 seconds (Fig 7b).

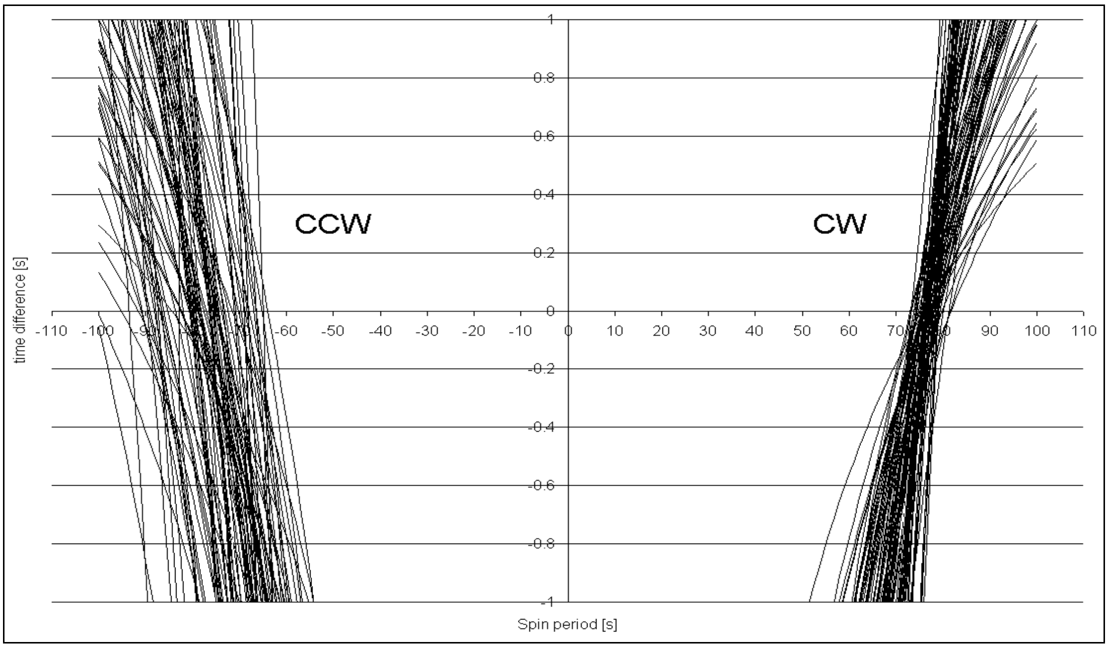


***Fig. 6:** Differences between T_{meas} and T_{sim} for 100 passes of phase A; CCW (left) and CW (right) spin directions have been simulated.*

Determination of Spin Direction

We define clockwise (CW) and counter clockwise (CCW) spin direction here as the spin of the spacecraft when looking on the LRR in pointing direction of GP-B. This spin direction of GP-B is a priori not known to us. To determine it using the kHz SLR data, both spin directions were simulated (Fig. 6).

The results in Fig. 6 indicate that GP-B spins CW, because the spread of the result here is much less than for the CCW simulation.

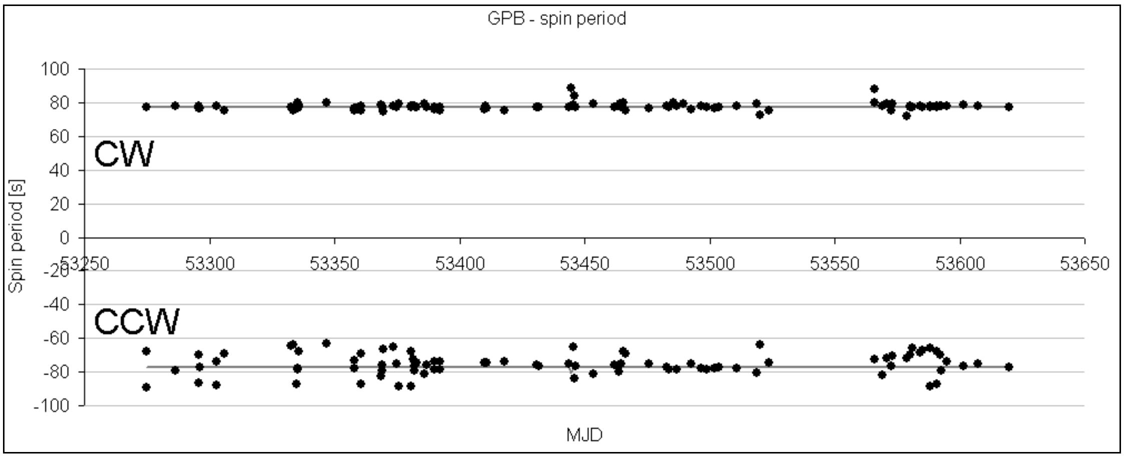


Fig. 7a: *GP-B spin period for 86 passes during phase A; positive values are for CW spin, negative for CCW spin assumed; solid lines at ±77.5 seconds indicate results of on-board spin measurements.*

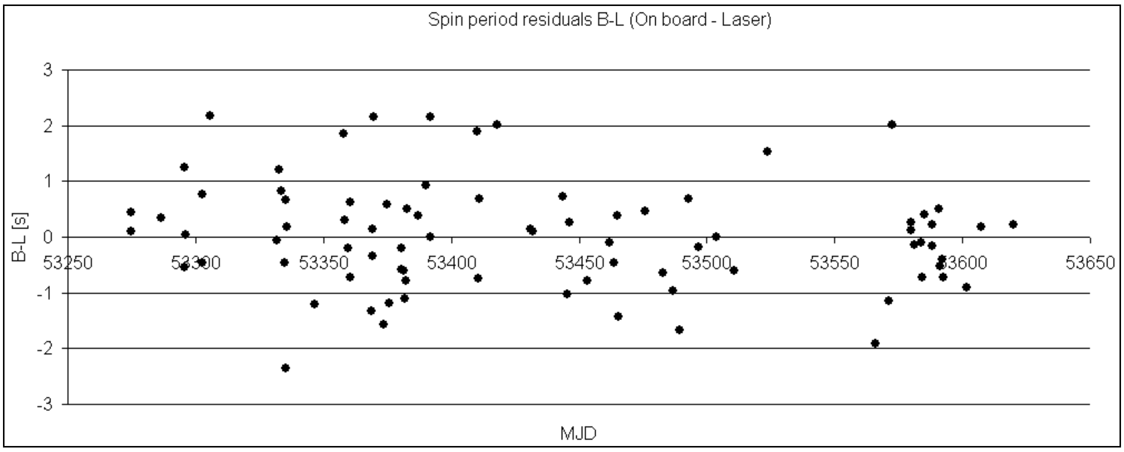


Fig. 7b: *Spin period for 86 passes of phase A with at least 5 peaks: differences to on-board spin period measurements; RMS of differences is 0.98 seconds.*

Determination of Spin Axis

Due to their periodically varying distances as seen by the SLR measurements, the eight laser reflectors generate specific patterns within the return data set, with a “modulation depth” depending on the incident angle between laser beam and GP-B’s axis (Fig. 8).

This change of the modulation depth within the pass can be used to evaluate the incident angle (Fig. 8, bottom) and thus at least one orientation angle of the satellite. However, this method proved to be more inaccurate than expected, mainly due to the limited resolution of the modulation depth determination; the instrumental jitter of about 3 mm RMS of the Graz kHz SLR system for GP-B is not really adequate to determine the modulation depth variations of 0 to 6 mm with sufficient accuracy.

Looking for a more suitable method to determine spin axis, the comparison between simulations and measurements once more proved to be appropriate. For this purpose, the returns from the 9th or central retro reflector, which are vaguely visible in a few passes, were used additionally (Fig. 9). Fitting a parabola to these returns, and determining the minimum value of the oscillations of the other 8 retro reflectors (Fig. 10), allows to fix the minimum distance between the upper and the lower curve (D), and the corresponding epoch time.

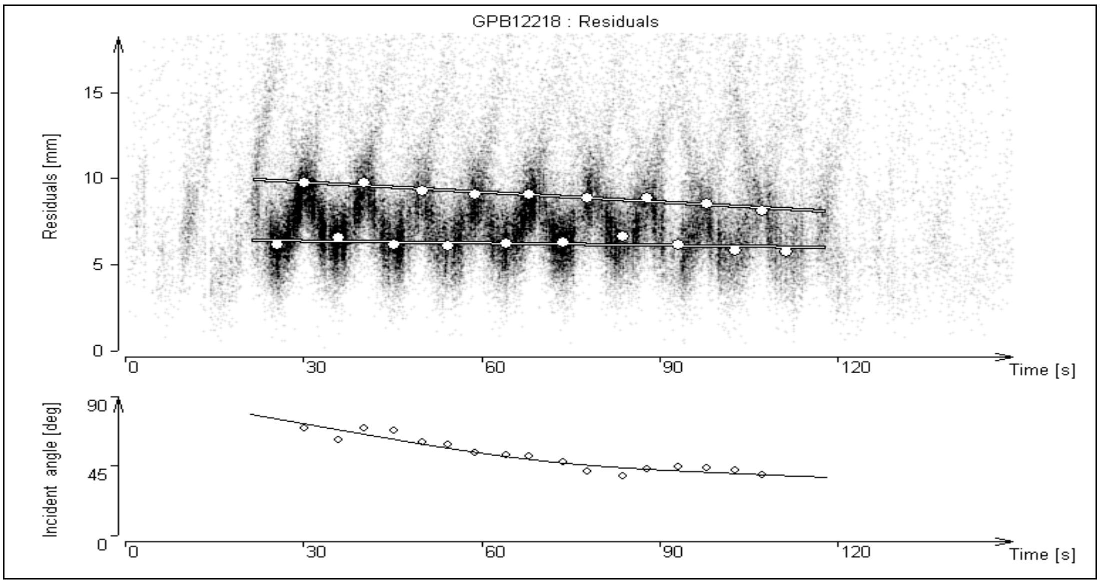


Fig. 8: *GP-B pass of DOY 122 / 2005: "Modulation depth" decreases during the pass (top); applying the known geometry of the retro reflectors, the incident angle of the laser beam can be determined (bottom).*

Running now simulations for this pass, spin axis longitude and colatitude (i.e. spin axis direction) were varied in steps of 1° each; for each spin axis direction, the spin period was calculated with the same method as described above. The goal was to find a combination spin period and spin axis direction, so that epoch time differences (between 9th retro parabola minimums of measurement and simulation) and range differences D (between simulations and measurements) are zero or close to zero.

Fig. 11 plots these differences between simulations and measurements; on the X-axis the differences in epoch time, on the Y-axis the differences in the distances D are shown; each line (set of points) represents solutions for different spin axis longitudes, and each point on these lines represents a solution for different spin axis colatitudes. The lowest line indicates a longitude of 320°, step size is 1°; zero for epoch and range differences means that the correct spin axis angles have been used in the simulation, as well as the correct inertial spin period; using this rough graph, the approximate longitude solution is between 340° and 341°, and the approximate colatitude between 73° and 74° (Fig. 11, left).

Using these values as boundaries for a more detailed simulation run with step sizes of 0.1°, we get about 341.4° for longitude, and 73.3° for colatitude, at an inertial spin rate of 77.42 seconds (Fig.11, right).

Two more GP-B passes were analyzed in this way, and the spin axis parameters determined; all results were coinciding with the on-board values with good accuracy (Table 1): standard deviation of the differences is 1.6° for colatitude, 1.77° for longitude, and 0.6 seconds for spin period.

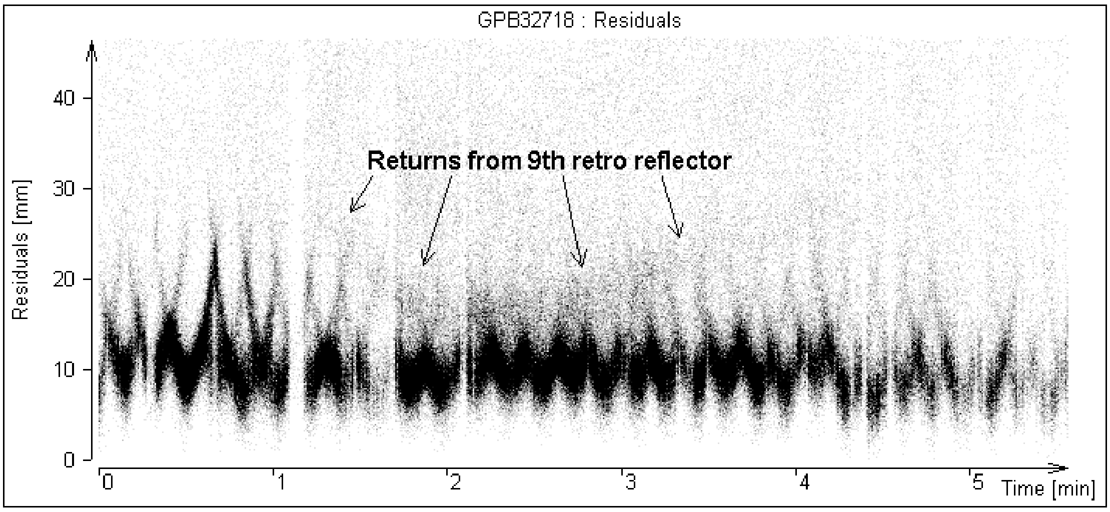


Fig. 9:. *GP-B pass of DOY 327/2005: Vaguely visible returns from 9th retro.*

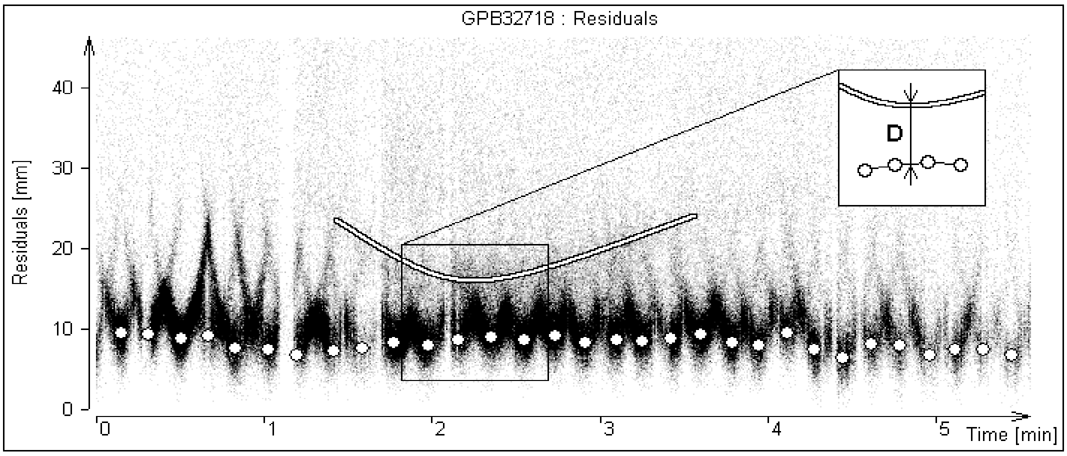


Fig. 10:.*Parabola fitted to 9th retro returns, gives epoch time and value of "D"*

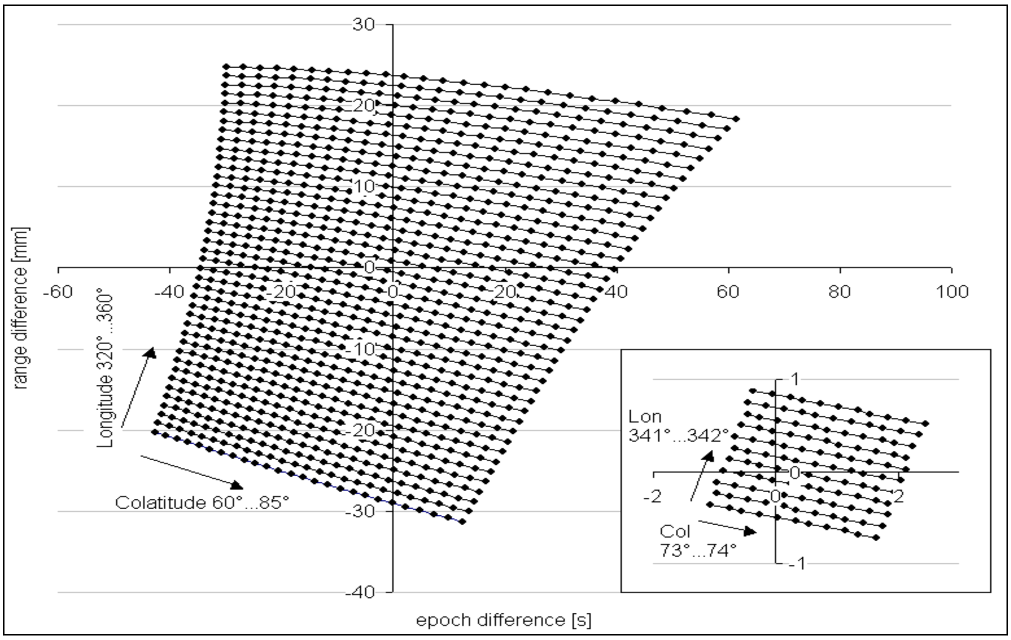
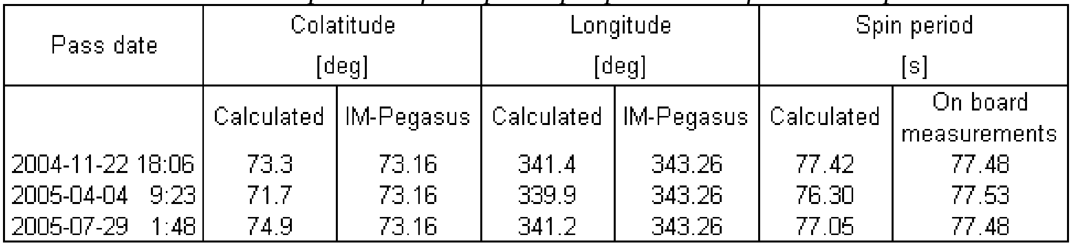


Fig. 11:.*Simulations for Longitude and Latitude vales of GP-B Spin Axis, varied in 1°-steps (left); same with 0.1° steps around ZERO (right)*

Table 1: *Comparison of complete spin parameters for 3 GP-B passes.*



Pass date	Colatitude [deg]		Longitude [deg]		Spin period [s]	
	Calculated	IM-Pegasus	Calculated	IM-Pegasus	Calculated	On board measurements
2004-11-22 18:06	73.3	73.16	341.4	343.26	77.42	77.48
2005-04-04 9:23	71.7	73.16	339.9	343.26	76.30	77.53
2005-07-29 1:48	74.9	73.16	341.2	343.26	77.05	77.48

Conclusions and Future Aspects

Using only kHz SLR data to derive spin parameters of satellites, opens completely new possibilities and areas for present and especially for future missions; larger separations between the individual elements of the retro reflector arrays automatically would increase the resulting accuracy. Suitable LRR geometries - to allow the identification of returns from single retro reflectors - enables complete spin axis determination from kHz SLR measurements. To get a more uniform distribution of returns from retro reflectors at different locations on the satellite, it would be easy to attenuate all echoes to the single photon level, resulting e.g. in the GP-B case in a much clearer identification of the 9th retro returns (Fig. 9, 10).

As more such kHz SLR stations will be operational in the very near future (Herstmonceux in the UK, SLR 2000 in USA), the availability of kHz SLR data sets will increase, allowing even more accurate spin parameters determination. As the satellite's payload for SLR is only a passive retro array, without any need for power supply or transmission bandwidth – and without major concerns about operational life time - , it might be a good main or backup device to obtain independently spin parameters of satellites, in addition to its main task of precise orbit determination via SLR.

Acknowledgment

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References

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[2] LRR Manufacturer Home Page: http://www.iteinc.net/

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